

# Advanced Logic - Homework 7

S3145034 (Group D)

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Consider the following set of default rules:

$$D = \left\{ \delta_1 = \frac{P \vee Q \quad R}{R}, \delta_2 = \frac{R \cdot S}{\neg P \vee Q}, \delta_3 = \frac{P \quad \neg Q}{S} \right\}$$

and initial set of facts:

$$W = \{P\}$$

Let  $T = (W, D)$

Recall that a formula  $\phi$  is a skeptical consequence of  $T$  if and only if  $\phi$  is true in every extension of  $T$ , while  $\phi$  is a credulous consequence of  $T$  if and only if  $\phi$  is true in at least one extension of  $T$ .

1 Is  $(\delta_1, \delta_2)$  a process? Explain your answer

**Answer:**  $(\delta_1, \delta_2)$  is a process, because for  $\Pi_1 = (\delta_1)$ , we have  $In(\Pi_1) = Th(\{P, R\})$ , and therefore  $pre(\delta_2) = R \in In(\Pi_1)$ , so  $\delta_2$  is applicable to  $In(\Pi_1)$

2 Is  $(\delta_1, \delta_2)$  closed? Explain your answer

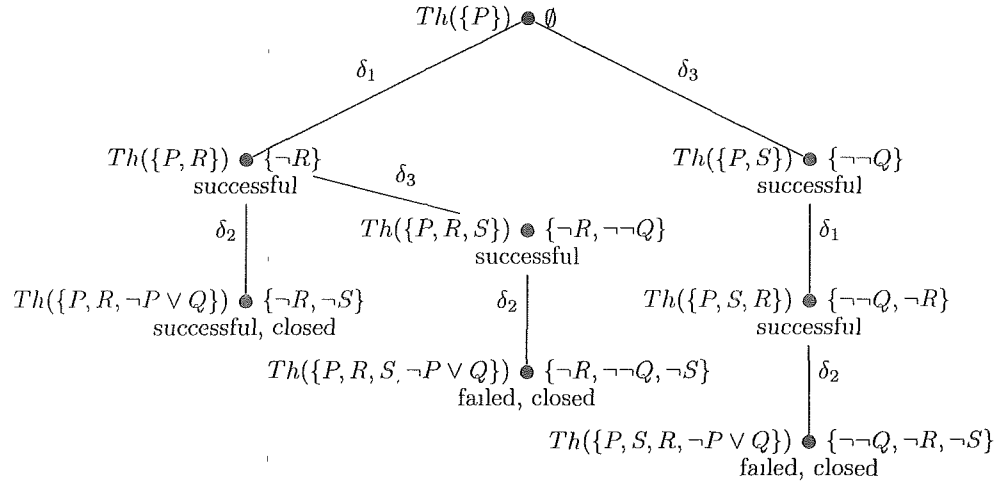
**Answer:** Yes,  $(\delta_1, \delta_2)$  is closed. We take  $\Pi = (\delta_1, \delta_2)$ , then  $In(\Pi) = Th(\{P, R, \neg P \vee Q\})$ . From  $P$  and  $\neg P \vee Q$ ,  $Q$  follows.  $\psi = \neg Q$ , the justification of  $\delta_3$ , now cannot be satisfied, so  $\delta_3$  is not applicable.

3 Is  $(\delta_1, \delta_2)$  successful? Explain your answer.

**Answer:** Yes,  $\Pi$  is successful. We have that  $In(\Pi) = Th(\{P, R, \neg P \vee Q\})$ ,  $Out(\Pi) = \{\neg S, \neg R\}$ . Neither element of  $Out(\Pi)$  occurs in  $In(\Pi)$ .

4 Draw the full process tree of the default theory  $T$ . Include the relevant In- and Out-sets and label the branches as appropriate: which ones are successful, failed, closed?

**Answer:**



5. Is  $S$  a credulous consequence of this theory? Explain your answer.

**Answer:** From the above process tree, we see that  $T$  only has a single extension, namely the one formed by the process  $(\delta_1, \delta_2)$ . Since  $S \notin In((\delta_1, \delta_2))$ , we conclude that  $S$  is not a credulous consequence of  $T$ .

6. Is  $P \wedge Q$  a skeptical consequence of this theory? Explain your answer.

**Answer:** We have that  $(P \wedge Q) \in In((\delta_1, \delta_2))$  (following from  $P$  and  $\neg P \vee Q$ ). Since  $(\delta_1, \delta_2)$  forms the only extension of  $T$ , we conclude that  $P \wedge Q$  is a skeptical consequence of  $T$ .